EXERCISE – III   SUBJECTIVE QUESTIONS

1. Find the coefficients

(i) x^7 in $\left(ax^2 + \frac{1}{bx}\right)^{11}$

Sol.

(ii) x^{-7} in $\left(ax - \frac{1}{bx^2}\right)^{11}$

Sol.

(iii) Find the relation between a and b, so that these coefficients are equal.
Sol.

2. If the coefficients of $(2r + 4)^{th}$, $(r - 2)^{th}$ terms in the expansion of $(1 + x)^{18}$ are equal, find r.

Sol.

3. If the coefficients of the r^{th}, $(r + 1)^{th}$ and $(r + 2)^{th}$ terms in the expansion of $(1 + x)^{14}$ are in A.P., find r.
Sol.

4. Find the term independent of x in the expansion of

(a) $\left[\sqrt{\frac{x}{3}} + \frac{\sqrt{3}}{2x^2}\right]^{10}$
Sol.

(b) $\left[\frac{1}{2}x^{1/3} + x^{-1/5}\right]^8$

Sol.

5. Find the sum of the series

$$\sum_{r=0}^{n}(-1)^r\,{}^nC_r\left[\frac{1}{2^r}+\frac{3^r}{2^{2r}}+\frac{7^r}{2^{3r}}+\frac{15^r}{2^{4r}}+\text{.........up to m terms}\right]$$

Sol.

6. If the coefficients of 2^{nd}, 3^{rd} and 4^{th} terms in the expansion of $(1 + x)^{2n}$ are in AP, show that $2n^2 - 9n + 7 = 0$.

Sol.

7. Given that

$(1 + x + x^2)^n = a_0 + a_1x + a_2x^2 + \ldots\ldots + a_{2n}x^{2n}$, find the values of

(i) $a_0 + a_1 + a_2 + \ldots.. + a_{2n}$;

(ii) $a_0 - a_1 + a_2 - a_3 \ldots\ldots. + a_{2n}$;

(iii) $a_0^2 - a_1^2 + a_2^2 - a_3^2 + \ldots + a_{2n}^2$

Sol.

8. If a, b, c and d are the coefficients of any four consecutive terms in the expansion of $(1 + x)^n$, $n \in N$, prove that $\frac{a}{a+b} + \frac{c}{c+d} = \frac{2b}{b+c}$.

Sol.

9. Find the value of x for which the fourth term in the expansion , $\left(5^{\frac{2}{5}\log_5\sqrt{4^x+44}} + \frac{1}{5^{\log_5\sqrt[3]{2^{x-1}+7}}}\right)^8$ is 336.

Sol.

10. Prove that :

${}^{n-1}C_r + {}^{n-2}C_r + {}^{n-3}C_r + \ldots\ldots\ldots\ldots + {}^{r}C_r = {}^{n}C_{r+1}$.

Sol.

11. (a) Which is larger : $(99^{50} + 100^{50})$ or $(101)^{50}$.

Sol.

(b) Show that ${}^{2n-2}C_{n-2} + 2.{}^{2n-2}C_{n-1} + {}^{2n-2}C_n > \frac{4n}{n+1}$, $n \in N$, $n > 2$.

Sol.

12. In the expansion of $\left(1 + x + \frac{7}{x}\right)^{11}$ find the term not containing x.

Sol.

13. Show that coefficient of x^5 in the expansion of $(1 + x^2)^5 . (1 + x)^4$ is 60.

Sol.

14. Find the coefficient of x^4 in the expansion of

(i) $(1 + x + x^2 + x^3)^{11}$

Sol.

(ii) $(2 - x + 3x^2)^6$

Sol.

15. Find numerically the greatest term in the expansion of

(i) $(2 + 3x)^9$ when $x = \frac{3}{2}$

Sol.

(ii) $(3 - 5x)^{15}$ when $x = \frac{1}{5}$

Sol.

16. Given $s_n = 1 + q + q^2 + ... + q^n$ and

$S_n = 1 + \frac{q+1}{2} + \left(\frac{q+1}{2}\right)^2 + ... + \left(\frac{q+1}{2}\right)^n$, $q \neq 1$.

Prove that ${}^{n+1}C_1 + {}^{n+1}C_2 \cdot s_1 + {}^{n+1}C_3 \cdot s_2 + ... + {}^{n+1}C_{n+1} \cdot s_n = 2^n \cdot S_n$.

Sol.

17. Prove that the ratio of the coefficient of x^{10} in $(1 - x^2)^{10}$ & the term independent of x in $\left(x - \frac{2}{x}\right)^{10}$ is 1 : 32.

Sol.

18. Find the term independent of x in the expansion of $(1 + x + 2x^3)\left(\frac{3x^2}{2} - \frac{1}{3x}\right)^9$.

Sol.

19. Let $(1 + x^2)^2 \cdot (1 + x)^n = \sum_{K=0}^{n+4} a_K . x^K$. If a_1, a_2 and a_3 are in AP, find n.

Sol.

20. If the coefficient of a^{r-1}, a^r, a^{r+1} in the expansion of $(1 + a)^n$ are in arithmetic progression then prove that $n^2 - n(4r + 1) + 4r^2 - 2 = 0$.

Sol.

21. If ${}^nJ_r = \dfrac{(1-x^n)(1-x^{n-1})(1-x^{n-2})......................(1-x^{n-r+1})}{(1-x)(1-x^2)(1-x^3)..........(1-x^r)}$, prove that ${}^nJ_{n-r} = {}^nJ_r$.

Sol.

22. Prove that $\sum_{K=0}^{n} {}^nC_K \sin Kx.\cos(n-K)x = 2^{n-1}\sin nx$.

Sol.

23. The expressions $1 + x$, $1 + x + x^2$, $1 + x + x^2 + x^3$, ..., $1 + x + x^2 + ... + x^n$ are multiplied together and the terms of the product thus obtained are arranged in increasing powers of x in the form of $a_0 + a_1x + a_2x^2 + ...$, then

(a) how many terms are there in the product

Sol.

(b) show that the coefficients of the terms in the product, equidistant from the beginning and end are equal.

Sol.

(c) show that the sum of the odd coefficients = the sum of the even coefficients = $\frac{(n+1)!}{2}$.

Sol.

24. Find the coefficients of

(a) x^6 in the expansion of $(ax^2 + bx + c)^9$

Sol.

(b) $x^2y^3z^4$ in the expansion of $(ax - by + cz)^9$.

Sol.

(c) $a^2b^3c^4$ d in the expansion of $(a - b - c + d)^{10}$

Sol.

25. If $\sum_{r=0}^{2n} a_r(x-2)^r = \sum_{r=0}^{2n} b_r(x-3)^r$ and $a_k = 1$ for all $k \geq n$, then show that $b_n = {}^{2n+1}C_{n+1}$.

Sol.

26. Find the coefficient of x^r in the expression of $(x+3)^{n-1} + (x+3)^{n-2}(x+2) + (x+3)^{n-3}(x+2)^2 + \ldots \ldots + (x+2)^{n-1}$.

Sol.

27. (a) Find the index n of the binomial $\left(\frac{x}{5}+\frac{2}{5}\right)^n$ if the 9th term of the expansion has numerically the greatest coefficient ($n \in N$).

Sol.

(b) For which positive values of x is the fourth term in the expansion of $(5 + 3x)^{10}$ is the greatest.

Sol.

28. Prove that $\frac{(72)!}{(36!)^2} - 1$ is divisible by 73.

Sol.

29. (a) Find the number of divisors of the number $N={}^{2000}C_1+2.\,{}^{2000}C_2+3\,{}^{2000}C_3+....+2000\,.{}^{2000}C_{2000}$

Sol.

(b) Find the sum of the roots (real or complex) of the equation $x^{2001} + \left(\frac{1}{2} - x\right)^{2001} = 0$.

Sol.

30. (a) Show that the integral part in each of the following is odd. $n \in N$.

(i) $(5 + 2\sqrt{6})^n$

Sol.

(ii) $(8 + 3\sqrt{7})^n$

Sol.

(iii) $(6 + \sqrt{35})^n$

Sol.

(b) Show that the integral part in each of the following is even. $n \in N$.

(i) $(3\sqrt{3}+5)^{2n+1}$

Sol.

(ii) $(5\sqrt{5}+11)^{2n+1}$

Sol.

31. If $(7+4\sqrt{3})^n$ = p + β where n and p are positive integers and β is a proper fraction show that
$(1 - \beta)(p + \beta) = 1$.

Sol.

32. If $(6\sqrt{6}+14)^{2n+1}$ = N and F be the fractional part of N, prove that $NF = 20^{2n+1}$ $(n \in N)$

Sol.

33. Prove that the integer next above $(\sqrt{3}+1)^{2n}$ contains 2^{n+1} as factor ($n \in N$)

Sol.

34. Let I denotes the integral part and F the proper fractional part of $(3+\sqrt{5})^n$ where $n \in N$ and if ρ denotes the rational part and σ the irrational part of the same, show that $\rho = \frac{1}{2}(I + 1)$ and $\sigma = \frac{1}{2}(I + 2F - 1)$.

Sol.

35. Prove that $\frac{{}^{2n}C_n}{n+1}$ is an integer, $\forall$ $n \in N$.

Sol.